

AP Calculus AB

Summer Packet – 2022

Name _____

Period _____

Score _____/140

Topic: Function Basics

If $f(x) = 4x - x^2$, find:

1.) $f(4) - f(-4)$

2.) $\sqrt{f\left(\frac{3}{2}\right)}$

3.) $\frac{f(x+h)-f(x)}{2h}$

If $V(r) = \frac{4}{3}\pi r^3$, find:

4.) $V\left(\frac{3}{4}\right)$

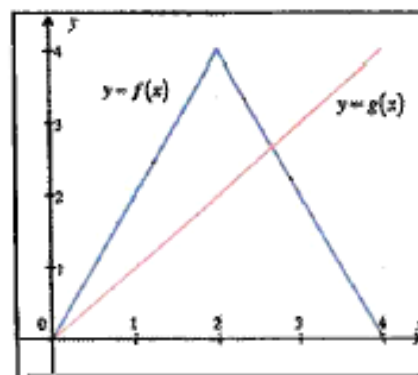
5.) $V(r+1) - V(r-1)$

6.) $\frac{V(2r)}{V(r)}$

If $f(x)$ and $g(x)$ are given in the graph, find:

7.) $(f - g)(3)$

8.) $f(g(3))$



If $f(x) = \begin{cases} -x, & x < 0 \\ x^2 - 1, & 0 \leq x < 2 \\ \sqrt{x+2} - 2, & x \geq 2 \end{cases}$, find:

9.) $f(0) - f(2)$

10.) $\sqrt{5 - f(-4)}$

11.) $f(f(3))$

Topic: Domain and Range

Find the domain of the following functions using interval notation:

12.) $f(x) = 3$

13.) $y = x^3 - x^2 + x$

14.) $y = \frac{x^3 - x^2 + x}{x}$

15.) $y = \frac{x-4}{x^2-16}$

16.) $f(x) = \frac{1}{4x^2-4x-3}$

17.) $y = \sqrt{2x-9}$

18.) $y = \log(x-10)$

19.) $y = \frac{\sqrt{2x+14}}{x^2-49}$

Find the range of the following functions:

20.) $y = x^4 + x^2 - 1$

21.) $y = 100^x$

22.) $y = \sqrt{x^2+1} + 1$

Topic: Even and Odd Functions

Show work to determine if the relation is even, odd, or neither. You may want to research how to determine evenness and oddness.

23.) $f(x) = 7$

24.) $f(x) = 2x^2 - 4x$

25.) $f(x) = -3x^3 - 2x$

26.) $f(x) = \sqrt{x+1}$

27.) $f(x) = \sqrt{x^2+1}$

28.) $f(x) = |8x|$

Topic: Special Factorization

Factor completely.

29.) $x^3 + 8$

30.) $x^3 - 8$

31.) $27x^3 - 125y^3$

32.) $x^4 + 11x^2 - 80$

33.) $ac + cd - ab - bd$

34.) $2x^2 + 50y^2 - 20xy$

35.) $x^2 + 12x + 36 - 9y^2$

36.) $x^3 - xy^2 + x^2y - y^3$

37.) $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

Topic: Linear Functions

Find the equation of the line in point-slope form, with the given slope, passing through the given point.

38.) $m = -7, (-3, -7)$

39.) $m = -\frac{1}{2}, (2, -8)$

40.) $m = \frac{2}{3}, (-6, \frac{1}{3})$

Find the equation of the line in point-slope form, passing through the given points.

41.) $(-3, 6), (-1, 2)$

42.) $(-7, 1), (3, -4)$

43.) $(-2, \frac{2}{3}), (\frac{1}{2}, 1)$

44.) Find k if the lines $3x - 5y = 9$ and $2x + ky = 11$ are a.) parallel and b.) perpendicular.

Topic: Solving Quadratic and Polynomial Equations

Solve each equation for x over the real number system.

45.) $x^2 + 7x - 18 = 0$

46.) $x^2 + x + \frac{1}{4} = 0$

47.) $2x^2 - 72 = 0$

48.) $12x^2 - 5x = 2$

49.) $20x^2 - 56x + 15 = 0$

50.) $81x^2 + 72x + 16 = 0$

51.) $x + \frac{1}{x} = \frac{17}{4}$

52.) $x^3 - 5x^2 + 5x - 25 = 0$

53.) $2x^4 - 15x^3 + 18x^2 = 0$

54.) If $y = x^2 + kx - k$, for what values of k will the quadratic have two real solutions?

Topic: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptote (if it exists) and the location of any holes.

55.) $y = \frac{x-1}{x+5}$

56.) $y = \frac{8}{x^2}$

57.) $y = \frac{2x+16}{x+8}$

58.) $y = \frac{2x^2+6x}{x^2+5x+6}$

59.) $y = \frac{x}{x^2-25}$

60.) $y = \frac{x^2-5}{2x^2-12}$

61.) $y = \frac{x^3}{x^2+4}$

62.) $y = \frac{x^3+4x}{x^3-2x^2+4x-8}$

63.) $y = \frac{10x+20}{x^3-2x^2-4x+8}$

64.) $y = \frac{1}{x} - \frac{x}{x+2}$ (Hint: Express with a common denominator)

Topic: Negative and Fractional Exponents

Simplify and write with positive exponents.

65.) -12^2x^{-5}

66.) $(-12x^5)^{-2}$

67.) $(4x^{-1})^{-1}$

68.) $\left(\frac{-4}{x^4}\right)^{-3}$

69.) $\left(\frac{5x^3}{y^2}\right)^{-3}$

70.) $(x^3 - 1)^{-2}$

71.) $(121x^8)^{\frac{1}{2}}$

72.) $(8x^2)^{-\frac{4}{3}}$

73.) $(-32x^{-5})^{-\frac{3}{5}}$

74.) $\frac{1}{4}(16x^2)^{-\frac{3}{4}}(32x)$

75.) $\frac{(x^2-1)^{-\frac{1}{2}}}{(x^2+1)^{\frac{1}{2}}}$

76.) $(x^{-2} + 2^{-2})^{-1}$

Topic: Complex Fractions

Eliminate the complex fractions:

77.) $\frac{\frac{5}{8}}{\frac{-2}{3}}$

78.) $\frac{4-\frac{2}{9}}{3+\frac{4}{3}}$

79.) $\frac{2+\frac{7}{2}+\frac{3}{5}}{5-\frac{3}{4}}$

80.) $\frac{x-\frac{1}{x}}{x+\frac{1}{x}}$

81.) $\frac{1+x^{-1}}{1-x^{-2}}$

82.) $\frac{x^{-1}+y^{-1}}{x+y}$

83.) $\frac{x^{-2}+x^{-1}+1}{x^{-2}-x}$

84.) $\frac{\frac{1}{3}(3x-4)^{-\frac{3}{4}}}{-\frac{3}{4}}$

85.) $\frac{2x(2x-1)^{\frac{1}{2}}-2x^2(2x-1)^{-\frac{1}{2}}}{(2x-1)}$

Topic: Inverses

Find the inverse of each of the following functions.

86.) $2x - 6y = 1$

87.) $y = ax + b$

88.) $y = 9 - x^2, x \geq 0$

89.) $y = \sqrt{1 - x^3}$

90.) $y = \frac{9}{x}$

91.) $y = \frac{2x+1}{3-2x}$

Find the inverse of each of the following functions and show that $f(f^{-1}(x)) = x$

92.) $f(x) = \frac{1}{2}x - \frac{4}{5}$

93.) $f(x) = x^2 - 4$

94.) $f(x) = \frac{x^2}{x^2+1}$

95.) Without finding the inverse, find the domain and range of the inverse to $f(x) = \frac{\sqrt{x+1}}{x^2}$

Topic: Adding Fractions and Solving Rational Equations

Combine the following fractions:

$$96.) \frac{2}{3} - \frac{1}{x}$$

$$97.) \frac{1}{x-3} + \frac{1}{x+3}$$

$$98.) \frac{5}{2x} - \frac{5}{3x+15}$$

$$99.) \frac{2x-1}{x-1} - \frac{3x}{2x+1}$$

Solve the equation for x .

$$100.) \frac{2}{3} - \frac{1}{x} = \frac{5}{6}$$

$$101.) \frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2-9}$$

$$102.) \frac{5}{2x} - \frac{5}{3(x+5)} = \frac{5}{x}$$

$$103.) \frac{2x-1}{x-1} - \frac{3x}{2x+1} = \frac{x^2+11}{2x^2-x-1}$$

Topic: Exponential Functions and Logarithms

Simplify the following:

$$104.) \log_2 \frac{1}{4}$$

$$105.) \log_8 4$$

$$106.) \ln \frac{1}{\sqrt[3]{e^2}}$$

$$107.) 5^{\log_5 40}$$

$$108.) e^{\ln 12}$$

$$109.) \log_{12} 2 + \log_{12} 9 + \log_{12} 8$$

$$110.) \log_2 \frac{2}{3} + \log_2 \frac{3}{32}$$

$$111.) \log_{\frac{1}{3}} \frac{4}{3} - \log_{\frac{1}{3}} 12$$

$$112.) \log_3 (\sqrt{3})^5$$

Solve the following:

$$113.) \log_5 (3x - 8) = 2$$

$$114.) \log_9 (x^2 - x + 3) = \frac{1}{2}$$

$$115.) \log(x - 3) + \log 5 = 2$$

$$116.) \log_5 (x + 3) - \log_5 x = 2$$

$$117.) \ln x^3 - \ln x^2 = \frac{1}{2}$$

$$118.) \log_2 (x - 1) + \log_2 (x + 3) = 5$$

$$119.) 3^{x-2} = 18$$

$$120.) e^{3x+1} = 10$$

$$121.) 8^x = 5^{2x-1}$$

Topic: Basic Right-Angle Trigonometry

Solve the following:

$$122.) \text{ If } \cos \theta = -\frac{5}{13}, \text{ in quadrant II, find } \sin \theta \text{ and } \tan \theta.$$

$$123.) \text{ If } \cot \theta = \frac{2\sqrt{10}}{3}, \text{ in quadrant III, find } \sin \theta \text{ and } \cos \theta.$$

State the quadrant in which each of the following is true.

$$124.) \sin \theta > 0 \text{ and } \cos \theta < 0$$


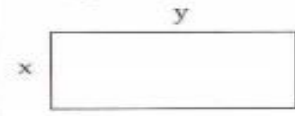
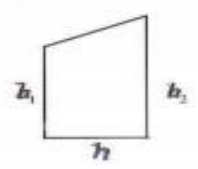
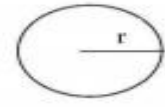
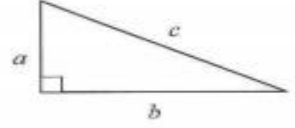
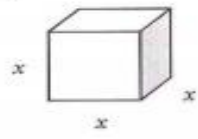
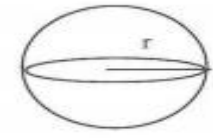
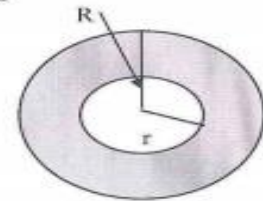
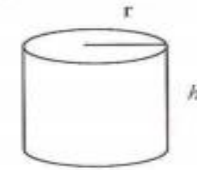
$$125.) \csc \theta < 0 \text{ and } \cot \theta > 0$$

$$126.) \tan \theta > 0 \text{ and } \sec \theta < 0$$

Topic: Geometry

You will use each of the following formula in AP Calculus AB.

127.-140.) Complete each of the following.

<p>Square</p> <div style="text-align: center;">  </div> <p>Perimeter = _____</p> <p>Area = _____</p>	<p>Rectangle</p> <div style="text-align: center;">  </div> <p>Perimeter = _____</p> <p>Area = _____</p>	<p>Trapezoid</p> <div style="text-align: center;">  </div> <p>Area = _____</p>
<p>Circle</p> <div style="text-align: center;">  </div> <p>Circumference = _____</p> <p>Area = _____</p>	<p>Triangle</p> <div style="text-align: center;">  </div> <p>Pythagorean Theorem (only good for right triangles) = _____</p> <p>Area (of any triangle) = _____</p>	<p>Cube</p> <div style="text-align: center;">  </div> <p>Volume = _____</p> <p>Surface Area = _____</p>
<p>Sphere</p> <div style="text-align: center;">  </div> <p>Volume = _____</p>	<p>"Washer"</p> <div style="text-align: center;">  </div> <p>Area of the shaded region = _____</p>	<p>Cylinder</p> <div style="text-align: center;">  </div> <p>Volume = _____</p>